Advertising effects in Sznajd marketing model

Christian Schulze

Institute for Theoretical Physics, Cologne University
D-50923 Köln, Euroland

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e-mail: ab127@uni-koeln.de

Abstract: The traditional Sznajd model, as well as its Ochrombel simplification for opinion spreading, are applied to marketing with the help of advertising. The larger the lattice is the smaller is the amount of advertising needed to convince the whole market.

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Science models like percolation have been applied to marketing by word-of-mouth [1] as well as advertising through mass media [2]. The present work applies the Sznajd model of consensus building [3] (see [4] for a review) to the same problem. How strong has the advertising to be in order to help one of two products to win the whole market even though initially this product is in the minority ?

In the Sznajd model as used here, initially a random fraction p of the sites of a square lattice are customers of product A, while the remaining fraction $1 - p$ of customers buy product B. At every iteration (random sequential updating) two neighbouring A sites convince their six neighbours to become also A sites. In the Ochrombel simplification, one A site suffices to convince its four neighbours [5]. After sufficiently many iterations in a finite lattice, all customers have settled onto one product. In the Sznajd model this product is the one which initially had a slight majority of customers, i.e. product A for $p > 1/2$ and product B for $p < 1/2$; for the Ochrombel modification a fraction p of lattices end up with only A sites, the others with only B sites.

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Figure 1: Number of successes, if advertising is switched on only after t_1 iterations, for 8 iterations. The different crosses refer to $t_2 - t_1 = 2, 4, 8, 16$ and 32, which hardly seems to matter.

Advertising is now included by assuming that at each iteration every site becomes an A site with probability ϵ. We define a success as meaning that all sites buy A, and a failure as meaning that inspite of the advertising all sites buy B for at least one iteration.

Figure 2: Number of successes in Ochrombel simplification with diffusion. From left to right the system size increases.

Diffusion makes the model more realistic by assuming [6] that only half of the sites are occupied; at each iteration each agent (= occupied site) moves into a randomly selected neighbour site if that neighbour is empty. ("Diffusion" is used here in the physics sense, not in that of Bass marketing theory [1]).

Feedback takes into account that advertising is diminished for already successful products. The fraction of A customers is called x, that of B customers is $y = 1 - x$. Then advertising produces an A site no longer with probability ϵ but with probability

$$\epsilon y(t)/y(t=0)$$

at iteration t.

Figure 3: Log-log plot of the level of advertising needed to convert half the failures to successes, versus linear lattice dimension L. The line has the slope -2.3.

This defines our advertising model which we now simulate with and without feedback, using the original Sznajd version, the Ochrombel simplification, and for Ochrombel also with diffusion. We used 1000 different samples for

each lattice size $L \times L$, $L = 31, 53, 71, 101, 301$. The initial fraction of A agents was always $p = 0.4$.

For the Sznajd model we found that without advertising nearly all samples ended with product B. With $\epsilon \sim 0.1$ and larger, all samples were successes, i.e. the advertising convinced everybody even though initially only a 40 percent minority was convinced. Equal numbers of failures and successes were found for $\epsilon \sim 0.04$ and 0.025 for $L = 31$ and 53 (not shown). Thus already a small fraction of advertising is sufficient to change nearly all samples from product B to product A.

We also simulated "ageing" by having advertising only for $t_1 < t < t_2$ Strong advertising $\epsilon = 0.5$ produced nearly always a success for t_1 up to 10 and was quite useless for $t_1 > 100$ at $L = 31$, Fig.1; for larger lattices the characteristic times are larger. The difference $t_2 - t_1 \sim 10$ was less important. In short, if a mass media campaign starts too late, then word-of-mouth propaganda through the standard Sznajd process has already cornered the market.

The Ochrombel simplification, that already a single site convinces its neighbours, is numerically much easier since no critical point at $p = 1/2$ occurs without advertising. Thus far less iterations are needed when failures and successes are nearly balanced. Without advertising we have 400 A fixed points and 600 B fixed points, and thus we ask how much advertising is needed to reduce the number of failures from 600 to 300. Fig.2 includes diffusion and shows that the needed ϵ increases from 0.0001 to 0.001 if L decreases from 101 to 31; the transition curves all have roughly the same shape. Without diffusion the needed advertising is slightly larger. Fig.3 shows that the needed advertising decreases roughly as $1/L^{2.3}$.

Quite similar results are obtained also with feedback (for both Sznajd and Ochrombel version), i.e. the model's results are quite robust. Again, the Ochrombel version requires less advertising than the original Sznajd version.
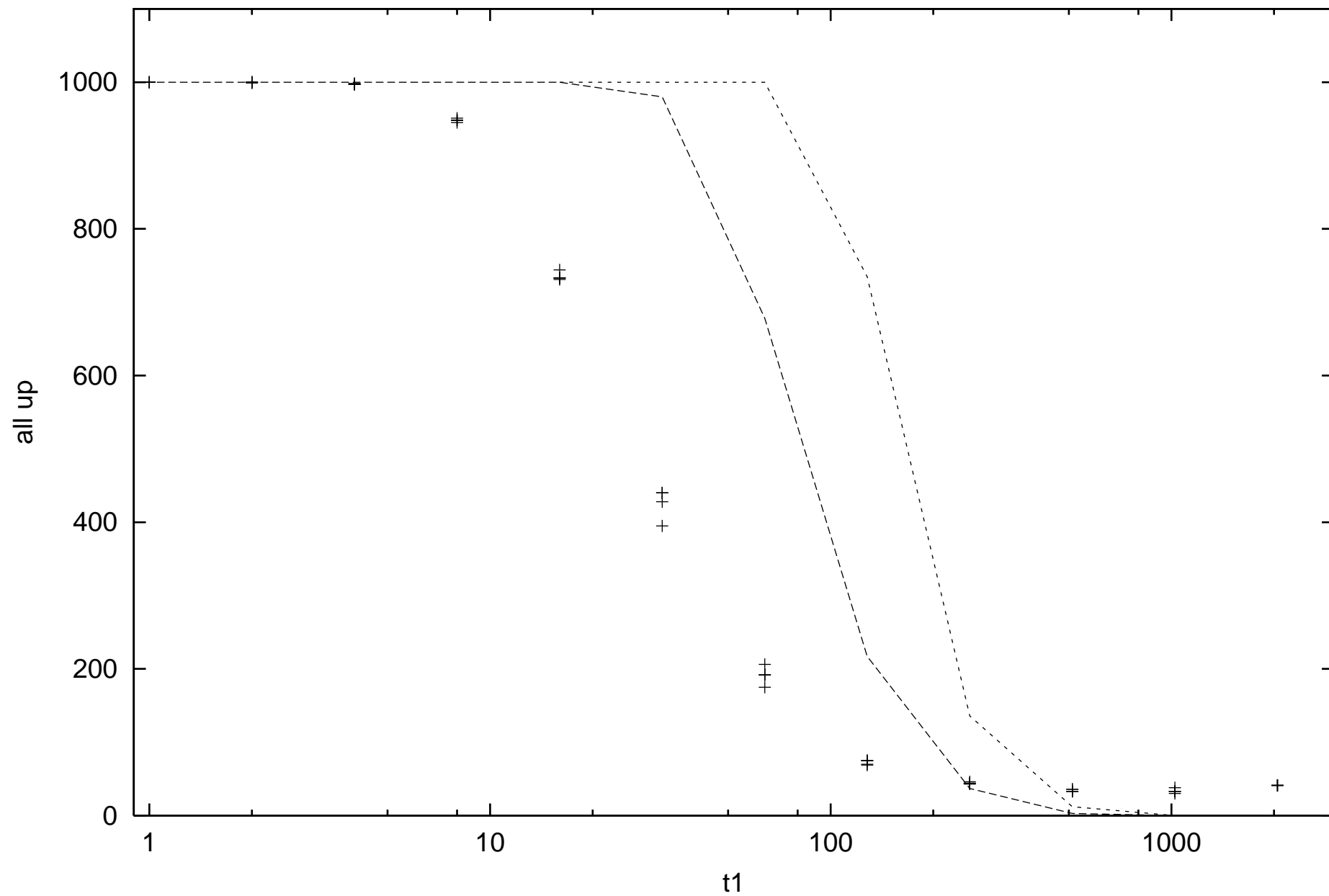
In summary, the Sznajd model and in particular its Ochrombel simplification are suitable to show successes and failures of advertising to convince a market.
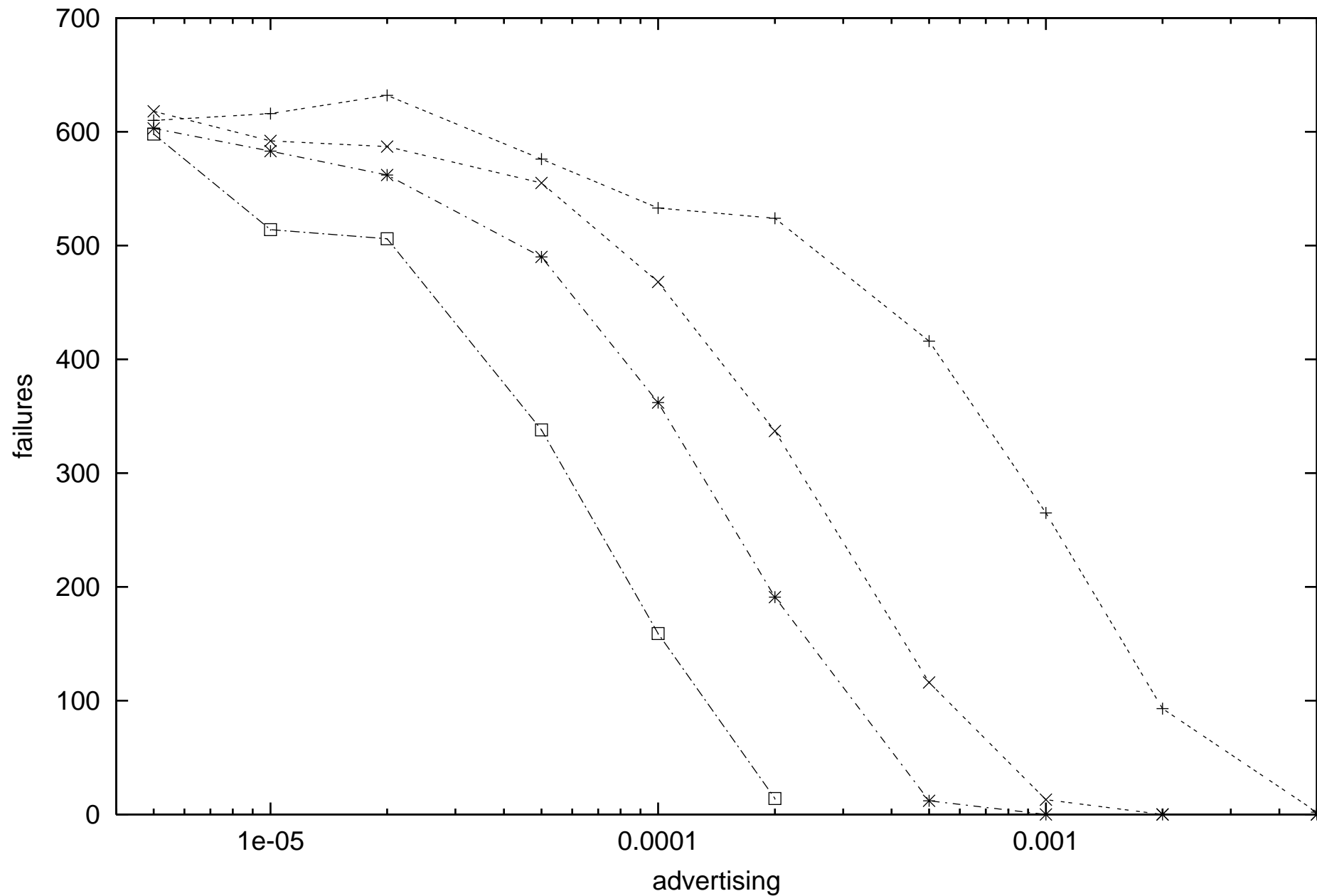
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1000 ageing runs, t2-t1 = 8, p=0.4, epsilon=0.5, L = 31 (+), 101 (left line), 301 (right line)

Single site convincing, diffusion, no feedback, 1000 samples at p=0.4, L = 31, 53, 71, 101

300 failures of 100; no diffusion (+,x), with diffusion (*,sq.), no feedback (+,*), with feedback (x,sq.)